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**Short Communication**

# Repose angles as a function of the supporting surface

J.T. CARSTENSEN, T.Y.-F. LAI, P. TOURE * and JANE SHERIDAN

*University of Wisconsin, School of Pharmacy, Madison, Wisc. 53706 and Hoffmann-la Roche, Nutley, N.J. 07110 (U.S.A.)*

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There have in the past been communications linking the repose angle, $\alpha$, of a monodisperse powder to fundamental particle parameters (cohesion, C, and internal friction, $\mu$). Hiestand (1966), to whom the basic correlation between $\alpha$, C and $\mu$ is attributed, cautions about experimental difficulties, and Pilpel (1971) finds it generally non-advisable to assess cohesiveness of powders based on angular measurements. However (Harwood and Pilpel, 1968), particles above 100 $\mu$m cannot be well assessed by shear and tensile strength measurements and hence repose angles still provide a sole means for some cohesive estimates. The limits in interpretation for large particles (granules) have also been treated in the literature (Carstensen, 1972, 1977; Carstensen and Chan, 1977, 1978). We wish to report here a point which never seems to have been taken into consideration when repose angle data are analyzed, and which may well be one of the experimental difficulties alluded to above, viz. the friction between the heap and the support.

Fig. 1A is a simplified picture of the force situation in a heap of monosized spherical particles in two dimensions. If the downward gravitational force (OQ) is unity, as shown in Fig. 1B, and is resolved at 30° to the horizontal, the two forces OP and OR will each be of magnitude 0.85 ($|OR| = |OP| = 0.5/(\cos 30°) = 0.85$). These forces are indicated by lines in Fig. 1A, so that, at the base there are 10 forces of the OP-type and 10 of the OR-type acting on the base. Each of these can be resolved along the horizontal and the vertical directions (Fig. 1C) to give forces of size $|OS| = 0.5$ in the vertical and $|OT| = 0.5 \cdot \cos 60 = 0.29$ in the two horizontal directions. The sum total of the OS-type forces is $20 \cdot 0.5 = 10$ (which is the gravitational force on the whole heap), and this is compensated for by reaction from the support.

The sum of the tangential forces of type OT is $10 \cdot 0.29 = 2.9$ in each direction and must be compensated for by the friction between the outermost spheres (W and W′) and the support.

The pictured resolution of the forces is based on a monodisperse heap of perfect spheres with a repose angle of 60°. It is restricted to two dimensions and is, therefore,

* On leave from Faculté de Pharmacie, Université de Paris-Sud, Châtenay-Malabry, France.

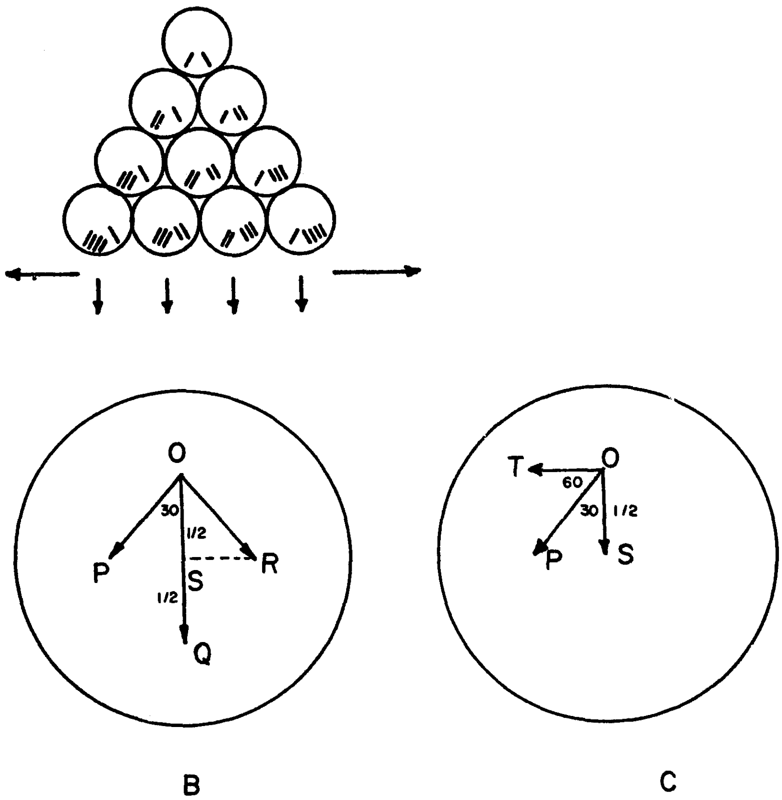


Fig. 1. A: simplified two-dimensional schematic drawing of a heap of spheres, each exerting a downward force of one arbitrary unit. B: resolution of gravitational force on top particle along 30° lines. C: resolution of resulting force on bottom, peripheral particle.

highly simplified. One simplification not mentioned is that of particle shape. In the limit for cubical particles of identical size, the horizontal component would be zero. It can therefore, at best, be stated that, with the simplifications mentioned, the horizontal component would be between zero and 2.9 units.

The figure, however, demonstrates the fact that friction between support and outer particles of the base of the heap are of importance. Hence, there should be some correlation between repose angle and frictional coefficient between material and support.

This has been investigated in the following fashion: a granulation of lactose and cornstarch, such as described by Carstensen and Chan (1977), was prepared. The 20/30 U.S.

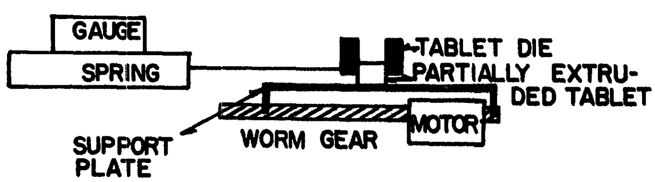


Fig. 2. Experimental set-up with I-Mass friction tester.

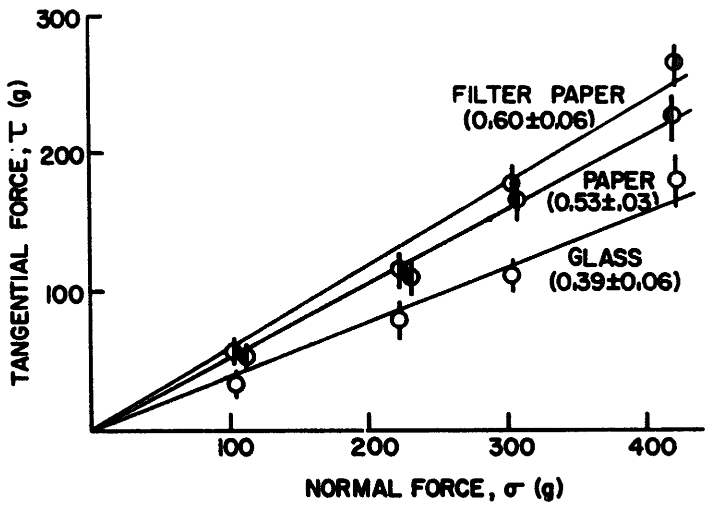


Fig. 3. $\tau$ versus $\sigma$ plots for partially ejected tablets on various supports on the support plate of the friction tester. Stainless steel data are omitted for graphical clarity.

mesh fraction (i.e. particle size of 715 μm on the average) was used for the study. Repose angle measurements were then performed as described by Carstensen and Chan (1977) but using the following materials for support of the heap: A, glass; B, stainless steel; C, bond paper and D, filter paper. Twenty-five measurements were made for each.

To obtain a measure of the frictional coefficient between the lactose granulation and these surfaces, the following procedure was used [1]: An I-Mass friction tester of the type shown in Fig. 2 [2] was used. This operates on the principle of measuring the drag force between two *surfaces* sliding one over the other. A support plate of polished stainless steel is attached to a worm gear and a motor, allowing it to move at constant velocity to the right. The second surface (e.g. plate) is held to a wire which is attached to a spring, the tension of which is recorded on a gauge. This is denoted tangential force ($\tau$ g force) in the following. The tangential force is then measured with different loads (normal forces, $\sigma$ g force). Plot of $\tau$ versus $\sigma$ should be linear and

$$\tau = \mu \cdot \sigma \qquad (1)$$

It follows that the slopes of the lines are equal to the frictional coefficients $\mu$.

The granulation, however, does not constitute a plane surface, so a 1 inch flat faced tablet was made at 15,000 p.s.i. on a hydraulic press. The tablet was partially expelled and the die attached to the string as shown in Fig. 2. $\tau$ versus $\sigma$ plots are linear as shown in Fig. 3. The frictional coefficient measured in this fashion is not the particulate friction between support and granule, but should somehow be a measure of it. The repose angles, as seen in Fig. 4, indeed are functions of the frictional coefficients obtained in this

[1] Direct measurement of friction between powder and support is possible, for instance by use of a Jenike cell. This was not attempted in this study.

[2] Visco-Tech, Cleveland, Ohio, U.S.A.

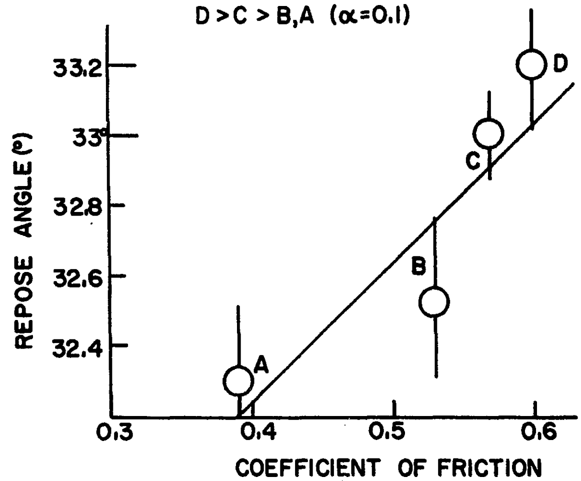


Fig. 4. Plot of $\alpha$ versus $\mu$. A, glass; B, stainless steel; C, bond paper; and D, filter paper.

fashion. Pairwise comparison shows that D (filter paper) has both a significantly higher frictional coefficient and repose angle than C (bond paper), which in turn possesses superior values to both glass and steel. The level of significance here is 90%. In addition, linear regression of $\alpha$ on $\mu$ gives a correlation coefficient of 0.98.

It would hence appear, as rationalized in Fig. 1, that the frictional force between support and base granules plays a significant part in the repose angle of a heap of granules on a plane surface.

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